Fluid mechanics / Lecture 2

Surface Tension :- the property of the liquid surface film to exert tension is called the surface tension. Surface tension is a measure of liquid tendency to take a spherical shape, caused by the mutual attraction of the liquid molecules.

- **Cohesion :- force attraction between the molecules of the same liquid.**
- **Adhesion :- force of attraction between the molecules of different liquids or between the liquid molecules and solid boundary containing the liquid.**

- Cohesion enables a liquid to resist very small tensile stress while adhesion enables a liquid to adhere to another body.
- Surface tension is due to cohesion between particles at the surface of liquid.
- Surface tension is the force exerted by the free surface of the liquid per unit length.
- Units :- Newton per meter (N/m)

- Dimension :- [MT^{-2}]
- The surface energy per unit area of interface is called surface tension.
- It is also expressed as work done per unit surface area.

$$\sigma = \frac{W \text{ (or) } E}{A} \, J/m^2$$

- As temperature increases, surface tension decreases

(because cohesion decreases).

- A (tensiometer) and (stalagmometer) are the experimental instruments used to measure the surface tension of liquid.

Pressure Inside a Water Droplet, Soap Bubble and a Liquid Jet

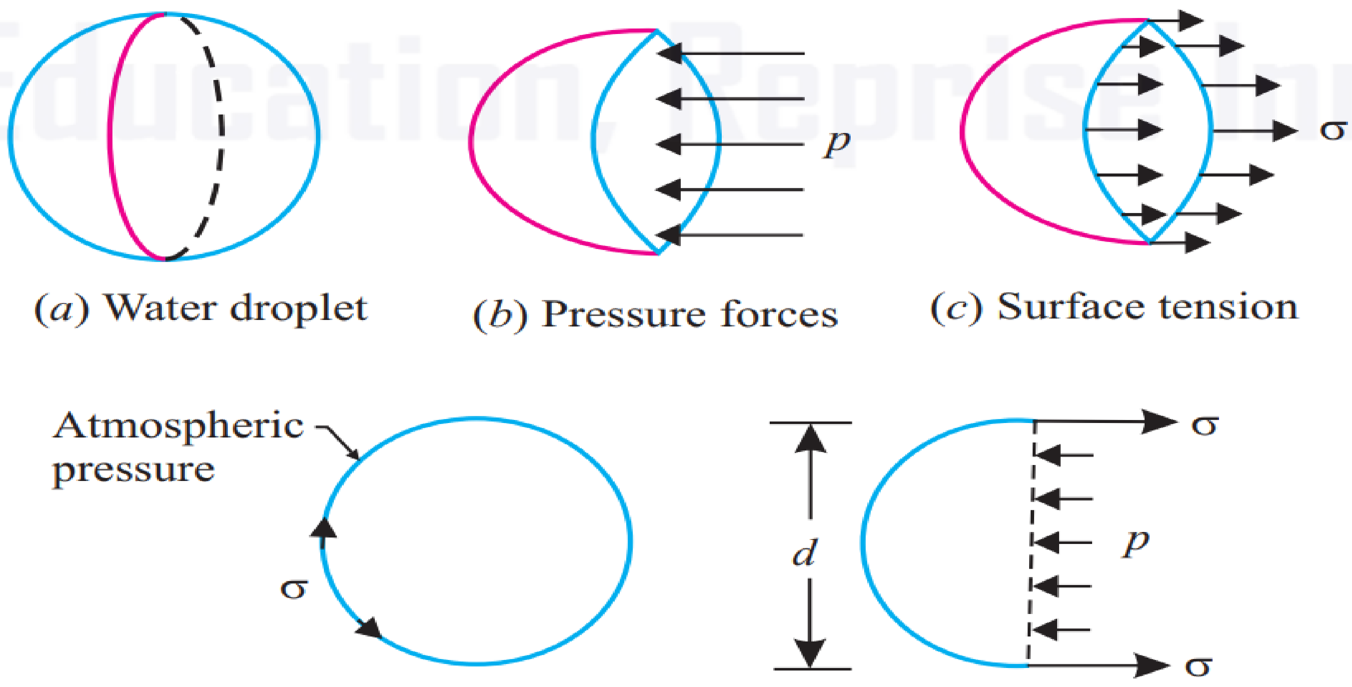
- **case1/ water droplet:-**

p= pressure inside the droplet above outside pressure.

d= diameter of the droplet and

σ= surface tension

from free body diagram, we have:-



(*a*) Water droplet (*b*) Pressure forces (*c*) Surface tension

(*d*) Free body diagram

i. Pressure force = $p \times \frac{\pi}{4} d^2$

ii. Surface tension force acting around the circumference $= \sigma \times \pi d.$

Under equilibrium conditions these two forces will be equal and opposite,

$$p \times \frac{\overleftarrow{\pi}}{4} d^2 = \sigma \times \vec{\pi} d$$

$$p = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d}$$

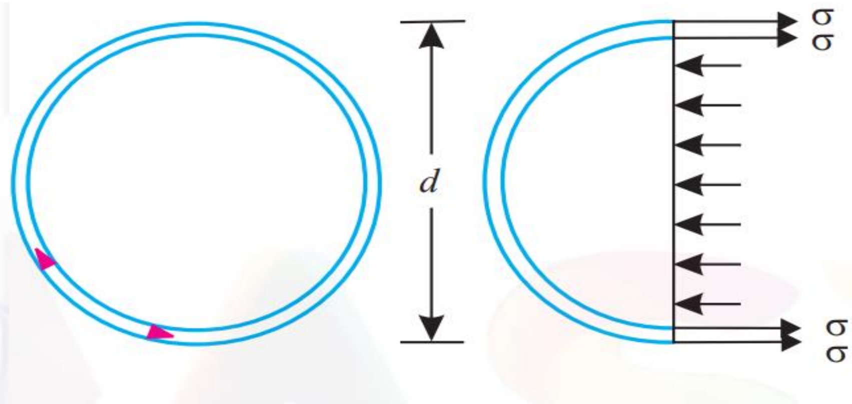
This eq. shows that with an increase in size of the droplet the pressure intensity decreases.

- Case II. Soap (or hollow) bubble: Soap bubbles have two surfaces on which surface tension σ acts.

From the free body diagram we have:-

$$\overleftarrow{p \times \frac{\pi}{4} d^2} = 2 \times (\overrightarrow{\sigma \times \pi d})$$

$$\therefore \quad p = \frac{2\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d}$$



Free body diagram

Since the soap solution has a high value of surface tension σ, even with small pressure of blowing a soap bubble will tend to grow larger in diameter (hence formation of large soap bubbles)

- Case III. A Liquid jet:

Let us consider a cylindrical liquid jet of diameter d and length l,

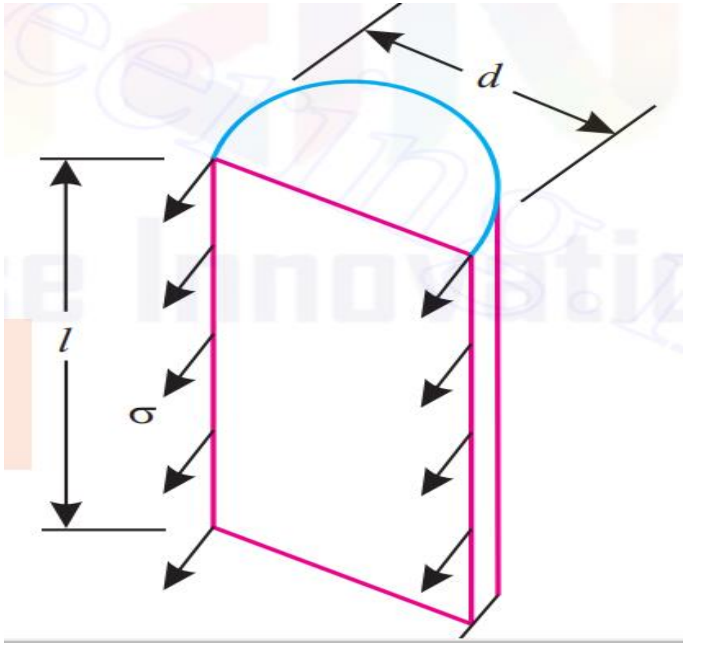
$$\text{Pressure force} = p \times l \times d$$

$$\text{Surface tension force} = \sigma \times 2l$$

Equating the two forces, we have:

$$p \times l \times d = \sigma \times 2l$$

$$\therefore \quad p = \frac{\sigma \times 2l}{l \times d} = \frac{2\sigma}{d}$$



- Example / If the surface tension at air-water interface is 0.069 N/m, what is the pressure difference between inside and outside of an air bubble of diameter 0.009 mm?

Solution/

Given: σ = 0.069 N/m; d = 0.009 mm

An air bubble has only one surface. Hence,

$$p = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.069}{0.009 \times 10^{-3}} = 30667 \text{ N/m}^2$$

$$= \mathbf{30.667\ kN/m^2\ or\ kPa}\ \textbf{(Ans.)}$$

Example [redacted] *A soap bubble 62.5 mm diameter has an internal pressure in excess of the outside pressure of 20 N/m². What is tension in the soap film?*

Solution. *Given*: Diameter of the bubble, $d = 62.5 \text{ mm} = 62.5 \times 10^{-3}$ m;

Internal pressure in excess of the outside pressure, $p = 20$ N/m².

Surface tension, σ:

Using the relation,
$$p = \frac{8\sigma}{d}$$

i.e.,
$$20 = \frac{8\sigma}{62.5 \times 10^{-3}} \quad \therefore \quad \sigma = 20 \times \frac{62.5 \times 10^{-3}}{8} = \mathbf{0.156\ N/m}\ \textbf{(Ans.)}$$

Problem [redacted] *The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.*

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3}$ m

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725$ N/m

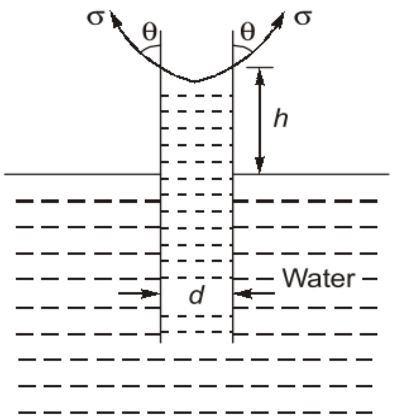
The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

or
$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

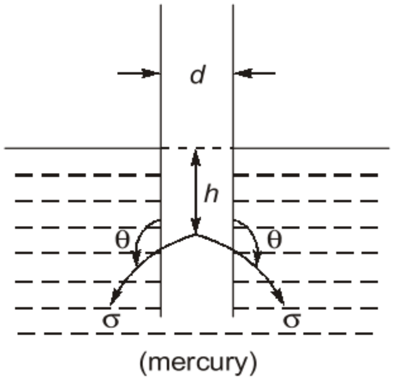
∴ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 = \mathbf{11.045\ N/cm^2}$. **Ans.**

Capillary :- the phenomenon of rise or fall of a liquid surface relative to the adjacent general level of liquid in small diameter tubes. The rise of liquid surface is designated as capillary rise and lowering is called capillary depression. Capillarity is due to both cohesion and adhesion.



Capillary rise



Capillary depression

- **Capillary rise:- for a liquid in contact with the surface, if adhesion predominates cohesion than the liquid will wet the surface with which it is in contact and tend to rise at the point of contact.**

- The free surface of the fluid will be concave upward and the contact angle (ϴ) will be less than 90^o. Example :- immersion of a glass tube in water.

- Capillary fall:- if for any liquid, in contact with a surface. Cohesion predominates the liquid will not wet the surface and the liquid surface will be depressed at the point of contact.
- The liquid surface will be concave downward and the angle of contact ϴ will be greater than 90^o.
- Such a phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid is known as capillarity.

$$h = \frac{4\sigma\cos\theta}{\gamma \cdot d}$$

h = Capillary height (rise/fall)

σ = Surface tension (N/m)

d = Diameter of tube (m)

γ = Specific weight of the liquid (N/m^3)

θ = Angle of contact between liquid and boundary

θ = 0° (water and glass) = 130° (mercury and glass)

Problem [redacted] *Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions $\sigma = 0.0725$ N/m for water and $\sigma = 0.52$ N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130°.*

Solution. Given :

Dia. of tube, $d = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m

Surface tension, σ for water $= 0.0725$ N/m

σ for mercury $= 0.52$ N/m

Sp. gr. of mercury $= 13.6$

$\therefore$ Density $= 13.6 \times 1000 \text{ kg/m}^3$.

(*a*) Capillary rise for water ($\theta = 0$)

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= .0118 \text{ m} = $ **1.18 cm. Ans.**

(*b*) For mercury

Angle of constant between mercury and glass tube, $\theta = 130°$

Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= -.004 \text{ m} = $ **– 0.4 cm. Ans.**

The negative sign indicates the capillary depression.

Problem [redacted] *The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension for water in contact with air = 0.0725 N/m.*

Solution. Given :

Capillary rise, $h = 0.2 \text{ mm} = 0.2 \times 10^{3} \text{ m}$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

Let dia. of tube $= d$

The angle θ for water $= 0$

Density (ρ) for water $= 1000 \text{ kg/m}^3$

Using equation (1.20), we get

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 0.2 \times 10^{-3} = \frac{4 \times 0.0725}{1000 \times 9.81 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.0725}{1000 \times 9.81 \times .2 \times 10^{-3}} = 0.148 \text{ m} = \mathbf{14.8 \text{ cm. Ans.}}$$

Thus minimum diameter of the tube should be 14.8 cm.

Compressibility :- all materials are compressible under the application of an external force. The compressibility of a fluid is defined as the change in volume of the fluid due to change in pressure. It is inversely proportional to the bulk modulus (K)

that: $K = \frac{\text{Change in Pressure}}{\text{Volumetric Strain}}$

$$\text{Change in Pressure} = \Delta p = P_{final} - P_{initial}$$

$$\text{Volumetric Strain} = \frac{V_{final} - V}{V} = \frac{\Delta V}{V}$$

$$K = -\frac{\Delta p}{\Delta V / V}$$

The –ve sign refer to the volume decrease as the pressure increase.

K can be expressed by another form as following:

$$K = \rho \frac{\Delta p}{\Delta \rho}$$

- Liquids are ordinary considered incompressible fluids since the change in volume (or density per unit mass) is so small that can be neglected, (K is constant). When (K) change, this means that the fluid is compressible such as in air (gasses in general).

Problem [redacted] *Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 N/cm² to 130 N/cm². The volume of the liquid decreases by 0.15 per cent.*

Solution. Given :

Initial pressure $= 70 \text{ N/cm}^2$

Final pressure $= 130 \text{ N/cm}^2$

$\therefore \quad dp$ = Increase in pressure $= 130 - 70 = 60 \text{ N/cm}^2$

Decrease in volume $= 0.15\%$

$$\therefore \qquad -\frac{d\forall}{\forall} = +\frac{0.15}{100}$$

Bulk modulus, K is given by equation (1.10) as

$$K = \frac{dp}{-\frac{d\forall}{\forall}} = \frac{60 \text{ N/cm}^2}{\frac{.15}{100}} = \frac{60 \times 100}{.15} = 4 \times 10^4 \text{ N/cm}^2. \textbf{ Ans.}$$

Example [redacted] *When the pressure of liquid is increased from 3.5 MN/m² to MN/m² its volume is found to decrease by 0.08 percent. What is the bulk modulus of elasticity of the liquid?*

Solution. Initial pressure = $3.5\ \text{MN/m}^2$

Final pressure = $6.5\ \text{MN/m}^2$

$\therefore$ Increase in pressure, $dp = 6.5 - 3.5 = 3.0\ \text{MN/m}^2$

Decrease in volume = 0.08 percent $\therefore\ -\dfrac{dV}{V} = \dfrac{0.08}{100}$

Bulk modulus (K) is given by:

$$K = \frac{dp}{-\frac{dV}{V}} = \frac{3\times10^6}{\frac{0.08}{100}} = 3.75 \times 10^9\ \text{N/m}^2 \text{ or } 3.75\ \text{GN/m}^2$$

Hence, K = **3.75 GN/m²** **(Ans.)**

Vapor pressure:-

All liquids have a tendency to evaporate or vaporize (i.e., to change from the liquid to the gaseous state). Molecules are continuously projected from the free surface to the atmosphere. These ejected molecules are in a gaseous state and exert their own partial vapour pressure on the liquid surface. This pressure is known as the vapour pressure of the liquid (pv). If the surface above the liquid is confined, the partial vapour pressure exerted by the molecules increases till the rate at which the molecules re-enter the liquid is equal to the rate at which they leave the surface. When the equilibrium condition is reached, the vapour pressure is called saturation vapour pressure (pvs). The following points are worth noting:

1. If the pressure on the liquid surface is lower than or equal to the saturation vapour pressure, boiling takes place.
2. Vapour pressure increases with the rise in temperature.
3. Mercury has a very low vapour pressure and hence, it is an excellent fluid to be used in a barometer.

Table 1.1. Summary of Fluid Characteristics

Sr. No.	Characteristics	Symbol	Definition	Dimensions	Units
1.	*Mass density*	ρ	Mass per unit volume, $\frac{m}{V}$	ML^{-3}	kg/m^3
2.	*Weight density* (or *specific weight*)	w	Weight per unit volume, $\frac{w}{V}$	FL^{-3}	N/m^3
3.	*Specific volume*	ν	Volume per unit mass $\frac{V}{m} = \frac{1}{\rho}$	L^3M^{-1}	m^3/kg
4.	*Specific gravity*	S	$\frac{\text{Specific weight of liquid}}{\text{Specific weight of pure water}}$ $= \frac{w_{liquid}}{w_{water}}$		
5.	*Dynamic viscosity*	μ	Newton's law: $\tau = \mu . \frac{du}{dy}$	FTL^{-2}	N.s/m^2 poise, centipoise

6.	*Kinematic viscosity*	ν	$\nu = \frac{\mu}{\rho}$	L^2T^{-1}	m^2/s stoke, centistoke
7.	*Bulk modulus*	K	$K = -\frac{\Delta p}{dV/V}$	FL^{-2}	N/m^2
8.	*Surface tension*	σ	Force per unit length	FL^{-1}	N/m
9.	*Vapour pressure*	p	$p_v = \frac{F}{A}$	FL^{-2}	N/m^2

Table 1.2. Properties of Some Common Fluids at 20°C and Atmospheric Pressure

Fluid	Mass density $\rho(kN/m^3)$	Specific weight $w(kN/m^3)$	Dynamic viscosity μ		Kinematic viscosity ν		Modulus of elasticity $E(N/m^2)$	Surface tension in contact with air, σ (N/m)	Vapour pressure (N/m^2)
			Poise	kg/ms	Stoke	m^2/s			
Air	1.208	0.01185	1.85×10^{-4}	1.85×10^{-5}	1.53×10^{-1}	1.53×10^{-5}	–	–	–
Benzene	860	8.434	0.007	7.00×1^{-4}	8.14×10^{-3}	8.14×10^{-7}	1.0356×10^{9}	0.0255	1.000×10^{4}
Castor oil	960	9.414	9.800	9.80×10^{-1}	1.00×10^{1}	1.00×10^{3}	1.441×10^{9}	0.0392	–
Carbon tetrachloride	1594	15.632	0.010	1.00×10^{-3}	6.04×10^{-3}	6.04×10^{-7}	1.104×10^{9}	0.0265	1.275×10^{4}
Ethyl alcohol	789	7.737	0.012	1.20×10^{-3}	1.52×10^{-2}	1.52×10^{-6}	1.118×10^{9}	0.0216	5.786×10^{3}
Glycerine	1260	12.356	8.350	8.35×10^{-1}	6.63	6.63×10^{-4}	4.354×10^{9}	0.0637	1.373×10^{-2}
Kerosene	800	7.845	0.020	2.00×10^{-3}	2.50×10^{-2}	2.50×10^{-6}	–	0.0235	–
Mercury	13550	132.880	0.016	1.60×10^{-3}	1.18×10^{-3}	1.18×10^{-7}	2.431×10^{10}	0.510	1.726×10^{-1}